Section Handout 3

Problem One: Order Relations

i. What three properties does a binary relation have to have to be a partial order?

ii. Consider the power set $\wp(A)$ of any set A. Prove that $\subseteq$ is a partial order over $\wp(A)$.

iii. Let | be the divisibility relation. We write $x \mid y$ if x divides y; that is, there exists some integer $k \in \mathbb{Z}$ such that $y = xk$. Prove that | is a partial order over $\mathbb{N}$, but that it is **not** a partial order over $\mathbb{Z}$.

Problem Two: Combining Relations

Suppose that $(A, \leq_A)$ and $(B, \leq_B)$ are ordered sets such that $\leq_A$ is a total order and $\leq_B$ is a total order. Consider the set $A \times B$. Define a relationship $\leq_{A\times B}$ on $A \times B$ such that $(a_1, b_1) \leq_{A\times B} (a_2, b_2)$ iff at least one of $a_1 \leq_A a_2$ and $b_1 \leq_B b_2$ is true.

i. Is $\leq_{A\times B}$ reflexive? If so, prove it. If not, give a counterexample.

ii. Is $\leq_{A\times B}$ antisymmetric? If so, prove it. If not, give a counterexample.

iii. Is $\leq_{A\times B}$ transitive? If so, prove it. If not, give a counterexample.

iv. Is $\leq_{A\times B}$ total? If so, prove it. If not, give a counterexample.

v. Based on your results from (i), (ii), (iii), and (iv), is $\leq_{A\times B}$ a total order?

Problem Three: Functions and Cardinality

Prove that for any sets A and B, $|A \times B| = |B \times A|$.

Problem Four: The Pigeonhole Principle

Prove that if 501 distinct natural numbers in the range 0 to 999 are chosen, there must some pair whose sum is exactly 999.